**Question 1: Suppose we wish to find the mean μ, for data that follows the normal distribution where the known data are {1, 5, 4, 10}. With two data items missing. Here and . Suppose that we initially guess . We then use this value for two missing values.**

**Answer:**

data1 = c(1, 5, 10, 4)

EM = function(data, N, k, m) {

mu = vector("numeric")

mu[1] = m

for (i in 1:1000) {

m = (sum(data) + (N - k) \* m) / N

p = mu[length(mu)] - m

if (abs(p) < 0.001)

break

else

mu[i] = m

}

cat("All mu:", mu, "\n")

cat("Final estimated mean (mu):", mu[length(mu)], "\n")

}

EM(data1, 6, 4, 3)

## All\_mu: 4.333333 4.777778 4.925926 4.975309 4.99177 4.997257 4.999086

## Final estimated mean (mu): 4.999086

**Question 2: Following data with two missing values follow Poisson distribution with mean 9 (use random seed = 10). Using Expectation-Maximization (EM) algorithm, estimate μ and number of iterations required to estimate μ (Use your exam roll as initial values).**

**Answer:**

set.seed(1810040)

mis = 2

N = 10

lambda2 = 9

pois2 = rpois(N, lambda2)

pois2[9:10] = NA

m = 10

mu = vector("numeric")

EM = function(x, N) {

mu[1] = m

for (i in 1:N) {

m = (sum(x, na.rm = T) + mis \* m) / N

p = mu[length(mu)] - m

if (abs(p) < 0.001)

break

else

mu[i] = m

}

cat("All mu:", mu, '\n')

cat("Final Mean (mu):", mu[length(mu)], '\n')

cat('No of Iterations:', i, '\n')

}

EM(pois2, 10)

All mu: 8.7 8.44 8.388 8.3776 8.37552

Final Mean (mu): 8.37552

No of Iterations: 6

**Question 3: Generate a random sample of size 20 from a Poisson distribution with mean 15. Use your examination roll as seed. Consider the last two values as missing. Using EM algorithm to estimate μ and σ. Comment on your estimates.**

**Answer:**

EM = function(N, lambda, mis) {

set.seed(1810040)

data = rpois(N, lambda)

data[19:20] = NA

m = round(sum(data, na.rm = T) / N)

mu = vector("numeric")

mu[1] = m

for (i in 1:1000) {

m = (sum(data, na.rm = T) + mis \* m) / N

if (abs(mu[length(mu)] - m) < 0.001)

break

else

mu[i] = m

}

data[19:20] = mu[length(mu)]

cat("Estimated Mean (mu):", mu[length(mu)], "\n")

cat("Estimated Standard Deviation (Sigma):", sd(data), "\n")

}

EM(20, 15, 2)

Estimated Mean (mu): 14.83315

Estimated Standard Deviation (Sigma): 3.391165

**Question 4: Generate a random number sample of size, 10 from an exponential distribution with mean 15. Using your examination roll as seed. Consider the 5th and 6th generated values as missing. Using EM algorithm,**

1. **Estimate the minimum variance unbiased estimate of the sample mean, and**
2. **An unbiased estimate of the variance of the sample mean. Comment on your findings.**

**Answer:**

EM = function(N, lambda, mis) {

set.seed(1810040)

data = rexp(N, rate = 1 / lambda)

data[5:6] = NA

m = round(N/sum(data, na.rm = T))

mu = vector("numeric")

mu[1] = m

for (i in 1:1000) {

m = (sum(data, na.rm = T) + mis \* m) / N

if (abs(mu[length(mu)] - m) < 0.001)

break

else

mu[i] = m

}

data[5:6] = mu[length(mu)]

cat("Answer (a):

Minimum Variance Unbiased Estimate:", mean(data), "\n", '\n')

cat("Answer (b):

Unbiased Variance of Sample Mean:", var(data)/N, "\n")

}

results = EM(10, 15, 2)

Answer (a):

Minimum Variance Unbiased Estimate: 20.35753

Answer (b):

Unbiased Variance of Sample Mean: 43.94035